

Chapter

5

Evaluating Performance

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Measuring Rates of Return

To evaluate the performance of a portfolio manager, you measure average portfolio returns. A rate of return (ROR) is a percentage that reflects the appreciation or depreciation in the value of a portfolio or asset.

PERAscope offers the following return methods:

- Dollar-Weighted Rate of Return
- Time-Weighted Rate of Return
- Income Return
- Principal Return

Dollar-Weighted Rate of Return

The Dollar-Weighted Rate of Return shows an average return of all the dollars in the portfolio for the period. It is also referred to as the Internal Rate of Return (IRR). If cash flows occur during the period, PERAscope uses the Internal or Dollar-Weighted Rate of Return for the period.

PERAscope calculates the Dollar-Weighted Rate of Return as follows:

Internal or Dollar-
Weighted Rate of
Return Calculation

$$MVB(1+i) + \sum_{k=1}^n F_k(1+i)^{\frac{ND-D_k}{ND}} = MVE$$

where

- MVB = The market value at the beginning of the period, including accrued income.
- i = The desired Internal or Dollar-Weighted Rate of Return.
- n = The number of cash flows during the period.
- F_k = The cash flow at time i during the period where contributions are positive cash flows and withdrawals are negative cash flows.
- ND = The number of days within the period.
- D_k = The number of days after the end of the previous period the F_k cash flow occurred.
- MVE = The market value at the end of the period, including accrued income.

For example, if the period being computed is the month of June and a flow occurred on June 5, D_k would be 5 and ND would be 30.

The Modified BAI calculation is an iterative process, where the *IRR* is obtained by selecting values for “*i*” and solving the equation until the result equals or is within a desired tolerance of *MVE*. The tolerance varies with the degree of precision selected during report generation.

Time-Weighted Rate of Return

For Time-Weighted Rates of Return, PERAScope uses a Modified BAI method that eliminates the impact of cash flows on performance calculations. Under this method, PERAScope calculates returns for time periods that have known market values at the beginning and end of the period. When calculating the rate of return for a given period, PERAScope determines the number (*n*) and timing of days within the period that have known market values. It then divides the period into *n+1* subperiods, so that each subperiod has known market values at the beginning and end of the period, and no known market values in between.

Subperiod rates of return are calculated differently depending on whether cash flows occur during the subperiod being measured. If there are cash flows, PERAScope uses the Internal or Dollar-Weighted Rate of Return calculation. See the “Dollar-Weighted Rate of Return” section on page 5-2 for detail.

If there are no cash flows during the period, the Time-Weighted Rate of Return is calculated as:

Time-Weighted Rate of Return Calculation

$$i = \frac{MVE}{MVB} - 1$$

where

- i* = The Time-Weighted Rate of Return for the subperiod.
- MVE* = The market value at the end of the subperiod, including accrued income.
- MVB* = The market value at the beginning of the subperiod, including accrued income.

Geometric Linking

Any consecutive set of subperiod returns can be linked together to create a Time-Weighted Rate of Return for a longer period. PERAscope links the subperiod rates of return geometrically as follows:

Geometric Linking

$$i = (1 + i_1)(1 + i_2) \dots (1 + i_{n+1}) - 1$$

where

i = The rate of return for the period.

i_k = The rate of return for subperiod 1, 2, through $n + 1$.

Annualized Returns

PERAscope reports the annualized return or return per year for a multi-year period as follows:

Annualized Return Calculation

$$AR = (1 + PR)^{1/n} - 1$$

where

AR = The Annualized Return.

PR = The Period Return.

n = The number of years in the period.

Income Rate of Return

The Income Rate of Return calculates the return from income net of expense. This is done by dividing the net income by the average market value:

Income Rate of Return Calculation

$$i_{Income} = \frac{Income}{AvgMV}$$

where

$$AvgMV = MVB + \sum_{k=1}^n F_k * \frac{ND - D_k}{ND}$$

i_{Income} = The sum of dividend income and coupon income for the period, plus the change in accruals from the beginning to end of the period, less expenses for the period.

MVB = The market value at the beginning of the period, including accrued income.

n = The number of cash flows during the period.

F_k = The cash flow at time i during the period.

ND = The number of days within the period.

D_k = The number of days after the end of the previous period the F_k cash flow occurred.

NOTE: If returns are being measured net of fees, then fees are also treated as an expense.

Principal Rate of Return

The Principal Rate of Return calculates the return based on changes in value of the assets. This is done by subtracting the income return from the total return:

Principal Rate of Return Calculation

$$i_{\text{Principal}} = i_{\text{Total}} - i_{\text{Income}}$$

where

i_{Total} = The total (time-weighted) return for the period.

i_{Income} = The income return for the period.

Daily Returns

Cash flows and market volatility can have a substantial impact on rate of return calculations. To ensure an accurate return, you can value your portfolios daily or whenever cash flows occur. This is known as daily valuation. Whether you value your portfolios monthly, daily, or whenever cash flows occur, PERAscope calculates the Dollar-Weighted, Time-Weighted, Income, and Principal Rate of Returns exactly the same way (that is, as described in the previous section on Measuring Rates of Return).

MPT Statistics

MPT Statistics are statistics common to a discipline known as Modern Portfolio Theory. PERAScope uses the following MPT statistics to evaluate risk and other factors relating to portfolios or assets:

- Geometric Mean
- Arithmetic Mean
- Beta
- Alpha
- R-Squared
- Standard Deviation
- Sharpe Ratio
- Treynor Ratio

Geometric Mean

PERAScope uses the geometric mean or average in all calculations involving the mean or average rate of return. It is the n th root of the product of n numbers and is used to measure the compound rate of return over time. Since the geometric mean represents the constant rate of return you would need to earn in each year to match actual performance over some past investment period, it is an excellent measure of past performance.

Geometric Mean Calculations

PERAScope calculates the geometric mean as follows:

$$\hat{i} = \sqrt[N]{\prod_{k=1}^N i_k}$$

where

- i = The rate of return.
- N = The number of data items.

The product of the monthly rates of return is further calculated as follows:

$$\prod_{k=1}^N i_k = i_1 * i_2 * i_3 * \dots * i_N$$

Alternately:

$$\hat{i} = \text{Exp} \left[\frac{1}{N} \sum_{k=1}^N \text{Ln}(1 + i_k) \right]$$

where

- i = The rate of return.
- Exp = The exponential function.
- Ln = The natural logarithm.

Arithmetic Mean

PERAScope uses the arithmetic mean to calculate standard deviation. See the [“Standard Deviation” section on page 5-14](#) for additional detail. The arithmetic mean or average is your standard, average calculation. If your focus is on future performance, you would use the arithmetic mean because it provides an unbiased estimate of the portfolio’s expected future return, assuming the expected rate of return does not change over time.

Arithmetic Mean Calculation

PERAScope calculates the arithmetic mean as follows:

$$\bar{i} = \frac{1}{N} \sum_{k=1}^N i_k$$

where

- \bar{i} = The rate of return.
- N = The number of data items.

Beta

Beta provides a measure of the tendency of an asset or portfolio's returns to respond to swings in the market. It is the slope of the security characteristic line obtained from linear regression. Linear regression is the process of finding the linear equation that best approximates the relationship between two variables. PERAScope calculates the beta as follows:

Beta Calculation

$$\beta = \frac{N \left[\sum_{k=1}^N (P_{xk} * M_{xk}) \right] - \left(\sum_{k=1}^N P_{xk} \right) \left(\sum_{k=1}^N M_{xk} \right)}{N \left(\sum_{k=1}^N M_{xk}^2 \right) - \left(\sum_{k=1}^N M_{xk} \right)^2}$$

where

- P_k = The k th instance of the portfolio's return.
- M_k = The k th instance of the market's return.
- N = The number of data items.
- P_{xk} = The k th instance of the portfolio's excess return ($P_k - R_k$).
- M_{xk} = The k th instance of the market's excess return ($M_k - R_k$).

This statistic reflects only the market-related portion of an asset's or a portfolio's risk. It is a narrower measure than standard deviation which reflects the total risk (market related and unique.) In general, the volatility of the relevant market index is considered to be 1.00, so a beta of 1.50 would indicate a volatility level 50% greater than that of the market. Since this statistic is relative to the market, betas for assets or portfolios with little or no correlation to the market are less significant. You may wish to consider R-Squared as a measure to determine the significance of this statistic.

Alpha

Alpha is a measure of risk adjusted performance that quantifies the difference between an asset's or portfolio's actual performance and the performance anticipated in light of the asset's or portfolio's risk (beta) and the market's (relevant market index) behavior. Alpha tells us how much better, or worse, a portfolio or asset did relative to what it was expected to do based on its risk posture. A positive alpha indicates a portfolio or asset's return has been more than commensurate with its risk posture.

Alpha is the Y intercept of the security characteristic line obtained from linear regression. Linear regression is the process of finding the linear equation that best defines the relationship between two variables.

NOTE: If the built-in linear regression function in your spreadsheet application uses the arithmetic mean, you will not get the same result. You must use the formula that follows since PERAScope uses the geometric mean.

Alpha Calculations

PERAScope calculates alpha as follows:

$$\alpha = \hat{P} - \beta * \hat{M}$$

where

\hat{P} = The geometric mean of the portfolio's excess return.

β = The slope of the security characteristic line obtained from the linear regression of the portfolio's excess return over the market's excess return.

\hat{M} = The geometric mean of the excess return of the benchmark.

PERAScope's Modern Portfolio Theory report contains an annualized alpha that is calculated as follows:

NOTE: This calculation assumes you are calculating alpha for a monthly return.

$$(\alpha)^{12}$$

R-Squared

R-Squared is the percentage of a portfolio's movement that can be explained by movement in its benchmark or in the market. This statistic indicates the percentage of an asset's or portfolio's risk which cannot be eliminated through further diversification. In precise percentage terms, this figure indicates just how closely an asset's or portfolio's performance variation paralleled the market over the same time period. Lower figures indicate less correlation with the market, and hence lower significance of the beta statistic. A relevant market index is used as a proxy for the market when measuring R-Squared.

R-Squared Calculation

PERAScope calculates R-Squared as follows:

$$R^2 = \frac{\left[N \left[\sum_{k=1}^N (P_{xk} * M_{xk}) \right] - \left(\sum_{k=1}^N P_{xk} \right) \left(\sum_{k=1}^N M_{xk} \right) \right]}{\sqrt{\left[N \left(\sum_{k=1}^N M_{xk}^2 \right) - \left(\sum_{k=1}^N M_{xk} \right)^2 \right] \left[N \left(\sum_{k=1}^N P_{xk}^2 \right) - \left(\sum_{k=1}^N P_{xk} \right)^2 \right]}}$$

where

- P_k = The k th instance of the portfolio's return.
- M_k = The k th instance of the market's return.
- R_k = The k th instance of the risk free benchmark.
- P_{xk} = The k th instance of portfolio's excess return ($P_k - R_k$).
- M_{xk} = The k th instance of market's excess return ($M_k - R_k$).
- N = The number of data items.

Sharpe Ratio

The Sharpe ratio or measure is the ratio of the portfolio's excess return to standard deviation. See the "Standard Deviation" section on page 5-14 for additional detail. It is calculated by dividing the geometric mean of the portfolio's excess return over a period by the standard deviation of returns over that period. Higher values are desirable and indicate a greater return per unit of risk.

Sharpe Ratio Calculation

PERAscope calculates the Sharpe ratio as follows:

$$\text{Sharpe} = \frac{\hat{P}}{\sigma_T}$$

where

\hat{P} = The geometric mean of the portfolio's excess return.

σ_T = The total return standard deviation.

Standard Deviation

Standard deviation provides a statistical measure of the daily, monthly, quarterly, or annual ups and downs of an asset's or portfolio's return. Money-market assets, which have stable asset values, have standard deviations near zero. Volatile, aggressive-growth portfolios can have standard deviations that are substantially higher.

Total Return Standard Deviation

Total return standard deviation measures the volatility of the total return of the portfolio. PERAscope calculates total return standard deviation as follows:

$$\sigma_T = \sqrt{\frac{\sum_{k=1}^N (P_k - \bar{P})^2}{N}}$$

where

- P_k = The total return for the k th period.
- \bar{P} = The arithmetic mean of the portfolio's returns over all included periods.
- N = The number of periods of returns included in the computation.

Excess Return Standard Deviation

Excess return standard deviation measures the volatility of the excess return of the portfolio. PERAscope calculates excess return standard deviation as follows:

$$\sigma_X = \sqrt{\frac{\sum_{k=1}^N (P_{xk} - \bar{P}_x)^2}{N}}$$

where

- P_{xk} = The excess return for the k th period.
- \bar{P}_x = The arithmetic mean of the portfolio's excess return over all included periods.

Treynor Ratio

The Treynor is a gauge of risk-adjusted performance that is calculated by dividing the excess return of a portfolio above the risk-free return by its beta. Higher values are desirable and indicate a greater return per unit of risk.

Treynor Ratio Calculation

PERAscope calculates the Treynor ratio as follows:

$$\text{Treynor} = \frac{\hat{P}}{\beta}$$

where

- \hat{P} = The geometric mean of the portfolio's excess return.
- β = The slope of the security characteristic line. This is usually interpreted as the level of undiversifiable risk of the portfolio.

